BARNHART

Determination of the Constants & Errors of a Three Inch Transit Zenith Telescope

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DETERMINATION OF THE CONSTANTS AND ERRORS OF A 3-INCH TRANSIT AND ZENITH TELESCOPE

BY

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Mr. Charles Anthony Barnhart

ENTITLED Determination of the Constants and Errors of a Three-inch Transit and Zenith Telescope.

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF Bachelor of Arts.

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Preface

It is the purpose of this short treatise to show the best methods of observation and computation in the use of a combined transit and zenith telescope for the accurate determination of time and geographical latitude. In order to obtain the desired accuracy for these quantities, an exact knowledge of all constants and errors of the instrument are necessary. This knowledge can only be obtained through an accurate determination of these constants and errors.

The writer has endeavored to make this treatise an essentially practical one and suited to the requirements of beginning work with a transit instrument. To that end, broken transits, curvature of a star's apparent path, etc, have not been treated as they constitute a part of more advanced study than is here contemplated.

An endeavor has been made to glean the best methods from recognized authorities, as well as to arrive at new methods of treatment. So far as the

writer knows, the details of the method by which he determines the collimation constant appear in print for the first time. The method seems to be more simple and desirable than those now in use. In several instances, slight improvements have been made to various methods already in general practice. An attempt has been made to introduce methods, due to others, which have not found their way into general practice as the method of computing the transit factors, entirely by means of natural functions by using Crelle.

The methods of reduction are intended to be exact to the extent that none of the value and precision of the observations will be sacrificed in the computations. Further refinement would lead to unreliable results and mislead the inexperienced beginner.

Comstock and Campbell have been the chief authorities consulted by the writer, both of whom have covered the field of practical astronomy in an admirable manner.

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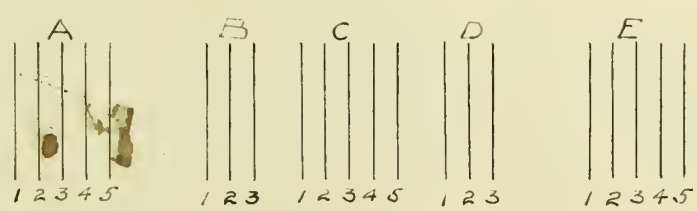
1. *Description of the Transit Instrument* — The essential parts of a transit are a *telescope* and a horizontal axis, to which the telescope is attached perpendicularly. The ends of this axis, *the pivots*, rest in angular notches called *wyes*. The straight line joining the centers of the pivots is the *rotation axis* and the straight line, passing through the optical center of the objective and intersecting the rotation axis perpendicularly is the *collimation axis*. If the telescope is revolved about the rotation axis, the collimation axis describes a plane, known as the *collimation plane*. This plane should be mounted in the plane of the meridian.

In the common focus of the objective and eye-piece is placed a system of wires called the *reticle*. These so-called wires are lines ruled upon glass, which have been filled with plumbago. The wires are arranged in the following manner;



The wires, always odd in number, are placed parallel to

the collimation plane and perpendicular to the rotation axis. The middle wire, w_3, is fixed as nearly as possible in the collimation plane. The times of transit of a star's image are observed over these wires. Two wires perpendicular to the above wires mark the center of the field of view. A micrometer wire is arranged so as to move parallel to the first set and as nearly as possible in the same plane.

The instrument is arranged so that its rotation axis can be reversed in azimuth by means of a lever. The pivots are raised carefully from the wyes by means of the lever. Keeping the lever in this position, the instrument is turned gently through 180°, and the pivots are lowered carefully until they rest firmly in the wyes. In this manner, the instrument may be reversed without appreciable jarring. The horizontal axis of the instrument is hollow and two electric light (one for clamp west and the other for clamp east), the positions of which are adjustable, are placed as nearly as possible in the rotation axis produced. The rays from a light pass through the pivot and fall upon a small mirror in the center of the telescope, which reflects them towards the eye piece, making the wires appear as dark lines in a bright field. The lights should be arranged so that they will not heat the instrument appreciably. The level, in this case a hanging level, is suspended from the horizontal axis. The mi-

crometer box can be rotated through 90° to make the movable wire either perpendicular or parallel to the rotation axis. A diagonal eye-piece enables the instrument to be used on zenith stars.

A coarse circle, with 30′ divisions, is used for setting the telescope at the approximate zenith distance of the star to be observed. With the telescope in this position, an accurate setting is obtained by using a smaller circle with 10′ divisions and with two verniers reading to the nearest 10″. These vernier arms are attached to a delicate level. After one of the verniers has been set at the proper reading for the star, the telescope is moved until the bubble plays. The star will then pass near the center of the eye-piece and can be made to pass between the two horizontal wires by turning the telescope in zenith distance by means of the slow-motion screw.

2. Theory of transit — If a transit instrument is in perfect adjustment, the rotation axis lies in the prime vertical; the horizon and the middle wire lie in the collimation plane. But since such an adjustment is impossible in practice, there are always three errors of adjustment that must be determined before any reliable work can be done with the instrument. These are (1), the azimuth constant a, the angle which the rotation axis makes with the prime vertical and which is positive when the west end

of the axis is too far south; (2) the level constant, b, the angle which the rotation axis makes with the horizon and which is positive when the west end of the axis is too high; and (3) the collimation constant, c, the amount by which the angle between the line of sight and the west half of the rotation axis exceeds 90°. The line of sight means the imaginary line passing through the optical center of the objective and the middle transit wire.

A fixed relation always exists between the above instrumental errors and the time, T, at which a star crosses a given wire. This relation is quite easily determined.

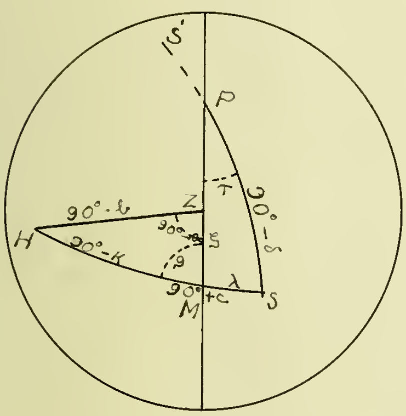


Fig. 1.

Fig. 1 represents a projection of the celestial sphere upon the plane of the horizon. Z is the projection of the zenith, P of the pole, H of the point in which the west half of the rotation axis pierces the sphere and S is the projection of a star observed just as it is crossing a wire, the angular distance from H of which is 90°+c. (In the figure a, b and c represent the errors discussed above). τ represents the hour angle of the star reckoned toward the east, δ is the star's declination, $90° - k$ is the arc HM, and λ is the distance of the star from the meridian measured along HS. It must be noted that the above symbols represent the actual magnitudes of the arcs and angles on the sphere.

From the spherical triangle MSP, we obtain,

$$\cos\delta \sin\tau = \sin\lambda \sin\vartheta$$

and in the triangle MHZ

$$\sin k = \sin b \cos \mathfrak{Z} + \cos b \sin \mathfrak{Z} \sin a$$

The sines may be replaced by arcs whenever a and b are sufficiently small that their cubes and higher powers are negligible (say, less than 5°). Assuming that the instrument with which we have to deal is adjusted, we may put $\sin\vartheta = 1$ and $\mathfrak{Z} = \Phi - \delta$, where Φ is the latitude of the observer. The above equations then would take the following forms

$$\tau = \lambda \sec\delta$$

$$k = b\cos(\Phi - \delta) + a\sin(\Phi - \delta)$$

From the figure

$$90° + c = 90° - k + \lambda$$

or

$$\lambda = c + k$$

$$= c + b\cos(\Phi - \delta) + a\sin(\Phi - \delta)$$

Therefore

$$\tau = c\sec\delta + b\cos(\Phi - \delta)\sec\delta + a\sin(\Phi - \delta)\sec\delta.$$

But τ is an east hour angle, and bears the following relation to T, the observed time, ΔT the chronometer correction and α the star's right ascension

$$T + \Delta T = \alpha - \tau$$

which gives by the elimination of τ, Mayer's equation of the transit instrument.

$$\alpha - T = \Delta T + \sin(\Phi - \delta)\sec\delta \cdot a + \cos(\Phi - \delta)\sec\delta \cdot b + \sec\delta \cdot c$$

or

$$\alpha - T = \Delta T + Aa + Bb + Cc.$$

where

$A = \sin(\phi - \delta)\sec\delta$, $B = \cos(\phi-\delta)\sec\delta$ and $C = \sec\delta$.

It should be noted that $(\phi - \delta) = z$, the zenith distance of the particular star, which is being observed.

The coefficients A, B and C are called transit factors, and when, as at an observatory, many observations are to be made at the same latitude, their values are tabulated with the declination as argument. From such tables, the value of these factors can be interpolated for any star.

We must also take into consideration the effect of diurnal aberration, which is to throw the star east of its true position. As a result, it is observed too late and the time of observation must be diminished by the quantity.

$$0''.31 \cos\phi \sec\delta \text{ or } 0^s.021 \cos\phi \cdot c$$

The formula giving the value of $\alpha - T$ was deduced for a star at upper culmination. Now, if A', B' and C' be the transit factors for a star at lower culmination

$$A + A' = 2\sin\phi, \quad B + B' = 2\cos\phi, \quad C + C' = 0.$$

A', B' and C' may be derived from the table of transit factors by making use of these equations. The factors are found to have the following algebraic signs for places in the northern hemisphere:

Factor	A	B	C
South of zenith	+	+	$\pm$
Zenith to pole	−	+	$\pm$
Below pole	+	−	$\mp$

All the factors have opposite signs above and below the pole due to the movement of the star in opposite directions. The double sign of C is due to the fact that the reversal of the instrument changes the sign of the collimation constant. It is customary to ignore this change of sign in c by changing the algebraic sign of C, whenever the instrument is reversed, that is

For clamp west $C(+c) = (+C)c$

For clamp east $C(-c) = (-C)c$.

The form of the transit factors in Mayer's equation of the transit instrument is not the best for the computation of these factors. They can be written in such a form that the computations may be done by means of Crelle, which is more rapid than by logarithms, since we shall have certain constant factors for all arguments of δ. As mentioned above, the form of the transit factors may be changed.

$$A = \sin(\phi-\delta)\sec\delta = \frac{\sin\phi\cos\delta - \cos\phi\sin\delta}{\cos\delta} = \frac{\cos\phi\sin\phi}{\cos\phi} - \cos\phi\tan\delta = \cos\phi(\tan\phi - \tan\delta)$$

$$B = \cos(\phi-\delta)\sec\delta = \frac{\cos\phi\cos\delta + \sin\phi\sin\delta}{\cos\delta} = \frac{\sin\phi\cos\phi}{\sin\phi} + \sin\phi\tan\delta = \sin\phi(\cot\phi + \tan\delta)$$

and as before

[1] Herman S. Davis. Popular Astronomy, 1902 p. 303.

$C = \sec \delta$

The computations may be arranged as shown in the table. It is to be noted that none of the intermediate steps in the computation need be discarded for the coefficients for Bessel's and Hansen's forms for reduction are given also.

Transit Factors

δ	tan δ	tan Q - tan δ	cot Q + tan δ	A	B	C
15°00′	0.268	0.574	1.455	0.439	0.937	1.035
20′	0.274	0.568	1.461	0.435	0.941	1.037
40′	0.281	0.561	1.468	0.429	0.946	1.039
16°00′	0.287	0.555	1.474	0.425	0.949	1.040
20′	0.293	0.549	1.480	0.420	0.953	1.042
40′	0.299	0.543	1.486	0.415	0.957	1.044
17°00′	0.306	0.536	1.493	0.410	0.962	1.046
30°00′	0.577	0.265	1.764	0.203	1.136	1.155
10′	0.581	0.261	1.768	0.200	1.139	1.157
20′	0.585	0.257	1.772	0.197	1.141	1.159
30′	0.589	0.253	1.776	0.194	1.144	1.161
40′	0.593	0.249	1.780	0.190	1.146	1.163
50′	0.597	0.245	1.7[illegible]	0.181	1.149	1.165
31°00′	0.601	0.241	1.7[illegible]8	0.184	1.152	1.167
55°00′	1.428	-0.586	2.61[illegible]	-0.448	1.684	1.743
05′	1.433	-0.591	2.62[illegible]	-0.45[illegible]	1.687	1.747
10′	1.437	-0.595	2.62[illegible]	-0.455	1.69[illegible]	1.751
15′	1.441	-0.599	2.62[illegible]	-0.458	1.693	1.754
20′	1.446	-0.604	2.633	-0.462	1.696	1.758
25′	1.450	-0.608	2.637	-0.465	1.698	1.762
30′	1.455	-0.613	2.642	-0.469	1.701	1.766
35′	1.460	-0.618	2.647	-0.473	1.705	1.769
40′	1.464	-0.622	2.651	-0.476	1.708	1.773
45′	1.469	-0.627	2.656	-0.480	1.711	1.777
50′	1.473	-0.631	2.660	-0.483	1.713	1.781
55′	1.478	-0.636	2.665	-0.487	1.716	1.78[illegible]
70°00′	2.747	-1.905	3.93[illegible]	-1.457	2.533	2.924
02′	2.752	-1.910	3.939	-1.461	2.536	2.928
04′	2.757	-1.915	3.94[illegible]	-1.465	2.540	2.933
06′	2.762	-1.920	3.947	-1.469	2.543	2.935
08′	2.767	-1.925	3.954	-1.473	2.546	2.942
10′	2.773	-1.131	3.960	-1.477	2.550	2.947
80°00′00″	5.671	-4.829	6.858	-3.694	4.417	5.758
00′30″	5.676	-4.834	[illegible].863	-3.698	4.420	5.764
01′00″	5.681	-4.839	[illegible]8	-3.7[illegible]	4.423	5.769
01′30″	5.686	-4.844	6.87[illegible]	-3.706	4.426	5.7[illegible]3
02′00″	5.691	-4.849	[illegible].87[illegible]	-3.709	4.430	5.778
[illegible]	5.695	-4.853	[illegible]	-3.711	4.432	5.782

sin Q = 0.644   tan Q = 0.842

cos Q = 0.765   cot Q = 1.18[illegible]

3 Value of One Revolution of the Micrometer Screw.

The value of one revolution of the micrometer screw must be determined in order to obtain the wire intervals, which are so important in time observations. Quite an accurate determination of this value may be made by observing a close circumpolar star. The telescope is directed so that the star is just visible within the limits of the field of view and will be carried through the middle portion of the field by its diurnal motion. The micrometer is set just in advance of the star, for convenience, with a micrometer reading of some multiple of five, and the time of transit of the star over the wire is recorded by means of a chronograph. Then the wire is moved forward five revolutions and a second transit observed. The observations are continued until the star has crossed the entire field.

In Fig. 2, let P be the pole, EP the observer's meridian, M′M″ the diurnal path of the star, AM′ one position of the micrometer wire and BM″ any other position of the wire. Now let r′ be the micrometer reading, τ′ the hour angle, and T′ the sidereal time when the star is at M′; and, let r″, τ″ and T″ be these quantities, respectively, when the star is at M″. The value of one revolution of the micrometer screw may be represented by R.

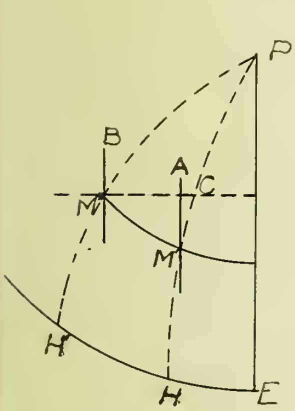


Fig. 2.

Through M″, describe an arc of a great circle perpen-

dicular to AM'. Then in the triangle CM"P

$$CM'' = (r''-r')R,\quad M''P = 90° - \delta,\quad CPM'' = T''-T' = T''-T';$$

and we can write

$$\sin [(r''-r')R] = \sin (T''-T') \cos \delta$$

or, since $(r''-r')R$ is always a small angle

$$(r''-r')R = \sin (T''-T') \cos \delta$$

and, when the star is not within 10° of the pole

$$(r''-r')R = (T''-T') \cos \delta$$

Then

$$R = \frac{(T''-T')\cos \delta}{r''-r'}$$

For extreme precision, the correction for refraction should be applied to the quantity $(r''-r')R$; but if the observations are made near the meridian, the correction will seldom be appreciable. If the value of a revolution in seconds of arc is desired, the above expression need only be multiplied by 15.

Example:- 50 Cassiopeiae and 36 H. Cassiopeiae were observed at upper culmination on Dec. 15, 1904 to determine the value of one revolution of the micrometer screw of the transit instrument. 50 Cassiopeiae was observed with the telescope clamp west and with the upper index of the micrometer and 36 H. Cassiopeiae with telescope clamp east and with the lower index.

r	$r''-r'$	T	$T''-T''$	r	$r''-r'$	T	$T''-T'$
10		$01^h\,51^m\,36^s.23$		40		$02^h\,22^m\,25^s.30$	
5		52^m 37.74		45		23^m 26.63	
50		53^m 37.[illegible]		50		24^m 28.10	
45		54^m 38.49		5		25^m 31.00	
40		55^m 39.00		10		26 33.26	
30	30	57^m 40.00	363.77	15		27 34.65	
25	30	58^m 40.[illegible]8	363.14	20	30	2[illegible] 37.52	$372^s.22$
20	30	59^m 41.00	363.20	25	30	29 38.79	372.16
				30	30	30 40.37	371.47
15	30	60^m 42.00	363.51	3[illegible]	30	31 43.50	372.50
				40	30	32 45.92	372.66
10	30	61^m 42.29	363.29	45	30	33 47.94	373.27

Mean 30 Mean 363.38 30 Mean 372.38

$$R = \frac{(T''-T')\cos\delta}{r''-r'}$$

Therefore for 50 Cassiopeiae

$$R = \frac{363.38 \times \cos 71^\circ 57' 50''}{30} = 3^s.7523$$

and for 36 H. Cassiopeiae

$$R = \frac{372.38 \times \cos 72^\circ 24' 15''}{30} = 3^s.7525$$

or the value of one revolution of the micrometer screw is $3^s.752$

4. Determination of the Wire Intervals — If the transit is provided with a micrometer in right ascension, this determination may be made quite easily. Theoretically, the micrometer wire should be set on each of the fixed wires in succession, but in practice this can not be done so as to give results of the desired accuracy. More accurate readings can be obtained by setting the micrometer wire on each side of the fixed wire and just in apparent contact with it. The mean of the readings in the two positions is the reading which the coincidence of the two wires should give. The difference of the micrometer readings on the side wires and the middle wire give the wire intervals in terms of one revolution of the micrometer screw, the value of which has already been obtained.

Example:— Two sets of micrometer readings were made in each of which the micrometer wire was placed in contact with each side of the fixed wires. The results are given in tabulated form on the next page. A, represents the first wire which a star approaching upper culmination would cross when the telescope is clamp west. The other wires are crossed in the order shown in the description of the transit.

Nov. 16, 1904

	A₁	A₂	A₃	A₄	A₅
	28.250	27.805	27.373	26.929	26.490
	28.330	27.887	27.453	27.008	26.569
	28.250	27.808	27.374	26.931	26.490
	28.331	27.883	27.449	27.009	26.573
Mean	28.290	27.846	27.412	26.969	26.531

	B₁	B₂	B₃
	24.730	24.288	23.852
	24.809	24.367	23.928
	24.730	24.287	23.847
	24.807	24.366	23.925
Mean	24.769	24.327	23.888

	C₁	C₂	C₃	C₄	C₅
	22.973	22.533	22.091	21.649	21.211
	23.052	22.609	22.168	21.722	21.284
	22.974	22.535	22.091	21.651	21.209
	23.051	22.608	22.166	21.724	21.289
Mean	23.013	22.571	22.129	21.687	21.248

	D₁	D₂	D₃
	20.327	19.892	19.452
	20.403	19.967	19.526
	20.327	19.893	19.452
	20.402	19.963	19.527
Mean	20.365	19.929	19.489

	E₁	E₂	E₃	E₄	E₅
	17.693	17.253	16.832	16.367	15.923
	17.769	17.328	16.906	16.447	16.009
	17.694	17.250	16.830	16.367	15.926
	17.768	17.332	16.908	16.445	16.006
Mean	17.731	17.291	16.869	16.407	15.966

Then, the difference of reading in terms of one revolution of the micrometer screw between each wire and the middle wire was taken. The first column in each case gives this difference and the second column gives the wire interval, which was obtained by multiplying the difference given in the first column by the value of one revolution of the micrometer screw, $3^s.752$.

A_1	6.161	23.12	E_5	6.163	23.12
A_2	5.717	21.45	E_4	5.722	21.47
A_3	5.283	19.82	E_3	5.260	19.74
A_4	4.840	18.16	E_2	4.838	18.15
A_5	4.402	16.52	E_1	4.398	16.50
B_1	2.640	9.90	D_3	2.640	9.91
B_2	2.196	8.24	D_2	2.200	8.25
B_3	1.759	6.60	D_1	1.764	6.62
C_1	0.881	3.32	C_5	0.881	3.31
C_2	0.442	1.66	C_4	0.442	1.66

5. The Value of a Level Division — The best method by which to determine the value of a level division would be to measure by means of a micrometer, the vertical angle through which a tube must be tipped in order to cause the bubble to pass over a given number of divisions of the scale. The necessary apparatus for this method was not available, so the level was attached to a theodolite with its plane approximately vertical.

Let the instrument be inclined until the vertical axis makes some angle γ, with the true vertical. Now, if the instrument is revolved slowly about its vertical axis, the bubble will move back and forth in the tube, and there will be two positions of the instrument, A_1 and A_2, for which the bubble stands at the middle of the scale. A slight revolution of the instrument to either side of these two positions will cause a correspondingly small movement of the bubble.

Fig. 3 which represents a portion of the celestial sphere adjacent to the zenith Z will be used to determine the relation of the bubble reading to the corresponding circle readings. V is the point in which the inclined vertical axis of the instrument pierces the sphere, and SV is the trace upon the sphere of the plane of the level-

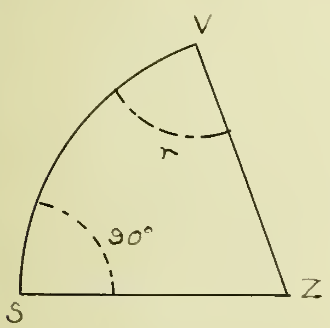


Fig. 3.

tube, which was attached to the instrument with its plane approximately vertical, before the instrument was tipped. As the instrument is rotated, the level-tube moves with it and the arc SV must revolve about V as a pivot. The amount of its rotation will be given by the consecutive readings of the azimuth circle. It is quite evident that the angle r of the figure for any particular reading A, is given by the equation

$$r = \frac{1}{2}(A_1 + A_2) - A$$

where $\frac{1}{2}(A_1 + A_2)$ is the circle reading for which SV and VZ are coincident.

It is quite easy to see that the bubble always stands at the point of its tube which is nearest the zenith. Therefore, the point in the figure corresponding to the middle of the tube may be found by drawing an arc of a great circle from Z perpendicular to SV. The intersection of this circle with SV will be the required point. Then, in the right spherical triangle ZVS, we have

$$\cos r = \frac{\tan \rho}{\tan y}$$

or

$$\tan \rho = \tan y \cos r$$

in which ρ measures the distance of the middle of the bubble from the fixed point, V.

To find how far the bubble will move when

the instrument is revolved slightly in azimuth, we need only to differentiate the above equation

$$\sec^2\rho \, d\rho = - \tan y \sin r \, dr$$

$$- b = \tan y \cos^2\rho \sin r \, dr.$$

Small finite increments of the respective quantities may be substituted in place of the differentials with the following result.

$$2 \, d (b' - b'') = \tan y \cos^2\rho \sin r (A' - A'')$$

where d represents the value of half a level division, and b', b'' are the readings of the middle of the bubble for the circle readings A' and A'' respectively.

This last equation may be employed to determine the value of d, but when sufficient care is taken to make the radius passing through the middle point of the scale parallell to the vertical axis of the instrument, the readings A_1 and A_2 will be so nearly $180°$ apart that we may put $r = 90°$ and $\cos\rho = 1$ for all positions of the bubble within the limits of the scale and obtain the much simpler relation

$$d = \frac{(A' - A'')}{2 (b' - b'')} \tan y$$

Example;— The axis of the instrument was deflected through an angle of 15' in the following manner. The instrument was carefully levelled and the telescope directed upon the steeple of a building. The telescope was moved through an angle of 15' by

means of the tangent movement to the vertical circle. Then the telescope was brought back to its former position by means of a footscrew. Care should always be taken that the footscrew will cause the telescope to move in the same plane as that in which it was inclined. If this is not done, the telescope can not be directed upon the point chosen when brought back by means of the footscrew. The axis of the instrument will be tipped through an angle of 15' as a result, as the error for such a small angle will be inappreciable.

A'−A" and b'−b" were derived from the readings of the horizontal circle, respectively. In making the observations, the bubble was brought near one end of the tube and the circle set to read some multiple of 5'. Then as the instrument was turned to each successive 5' reading, the readings of the extremities of the bubble were recorded until the bubble had traversed the entire length of the tube, after which the movement of the circle was reversed and readings taken for the same circle settings as before.

It is quite evident that the divisor 2(b'−b") is equivalent to 2b'−2b", and since b is the reading for the middle of the bubble, 2b is equal to the sum of the readings of the ends of the bubble. The differences 2b'−2b" were obtained by substracting the first b from the first

one following the middle of the set, the second b from the second one from the middle, etc.

Dec. 21, 1904

Circle Reading	Bubble Forward e	b	Backward e	b	Σb	$2(b'-b'')$
20	93.5	50.7	93.1	49.5	143.4	
25	90.8	47.9	91.4	47.8	138.9	
30	88.9	46.0	89.4	45.9	135.1	
35	86.8	43.9	87.3	44.0	131.0	
40	84.6	41.5	85.9	42.3	127.2	
45	83.0	39.9	84.6	41.2	124.3	
50	81.3	38.2	83.0	39.3	120.9	
55	80.1	37.0	81.0	37.7	117.9	
60	78.0	34.9	79.4	36.0	114.2	
65	76.0	33.0	71.0	33.5	109.7	
70	73.6	30.7	75.8	32.1	106.1	
75	72.2	28.9	73.6	30.0	102.4	
80	70.1	27.4	71.4	28.0	98.7	44.7
85	68.6	25.3	70.1	26.9	95.5	43.4
90	68.0	24.8	67.7	24.2	92.3	42.8
95	66.1	21.9	66.1	22.7	87.9	43.1
100	64.3	21.2	64.2	20.8	85.3	41.9
105	61.8	18.7	62.1	18.8	80.1	43.6
110	61.2	18.0	60.7	11.3	78.6	42.3
115	59.3	16.0	58.6	15.2	74.5	43.4
120	51.7	14.5	56.9	13.5	71.3	42.9
125	56.7	13.4	54.8	11.6	68.3	41.4
130	54.0	10.6	53.0	9.0	63.3	42.8
135	52.5	9.2	51.2	8.1	60.5	41.9

Mean 42.85

$Y = 15'$ $\log(A' - A'') = 3.5563$ $\log d = 9.5641$

$A' - A'' = 60' = 3600''$ $\operatorname{colog} 2(b' - b'') = 8.3685$ $d = 0''.367 = 0^s.024$

$\log \tan Y = 7.6398$ Level division $= 2d = 0''.734 = 0^s.049$

6 <u>Determination of the Level Constant</u> — As already defined, the level error, b, is the angle of elevation of the rotation axis above the western horizon. A repetition of the definition was deemed advisable here in order to make the method of this determination more clear. As the zero of the scale was at one end of the tube, and the numbers increased continuously to the other, we shall consider that case. All other cases would be treated in a similar manner.

If the level be placed on a truly horizontal line, the center of the bubble will not be at zero, because of the nonadjusted of the level; for instance, the case when one leg is longer than the other, etc. Suppose the center is n divisions from the zero mark, then the error of the level is $\mathfrak{I} nd$, where $\mathfrak{I} d$ represents one level division. Now, let the level be placed in a position forming an angle b with the horizon and let the reading of the west end of the bubble be w and of the east end, e. Then, the elevation of the west end of the line is given by

$$b = \tfrac{1}{2}(w+e)\mathfrak{I} d \mp \mathfrak{I} nd$$

If the level is reversed

$$b = -\tfrac{1}{2}(w'+e')\mathfrak{I} d \pm \mathfrak{I} nd$$

for the west end of the tube would form an angle of opposite sign with the horizon. Combining these two

values of b, we have

$$b = \frac{1}{2}\left[(w-e')+(e-w')\right]d$$

A positive value of b will be obtained whenever the west end is higher than the east end. It is to be noticed that the error of the level is eliminated by reversing the instrument. Whenever it is possible the level should be read several times, the same number of readings being made in each position — clamp west and clamp east.

Example:— It was necessary to determine b, in some work done on Dec. 5, 1904

	w	e	w	e
Clamp west	90.3	16.2	89.1	15.0
Clamp east	9.0	83.3	9.0	85.0
	7.1		5.05	

Mean +6.08

$$b = 6.08 \times 0^s.024 = +0.146$$

But the level is applied to the outer surface of the cylindrical pivots and does not give the inclination of the axis, which passes through their centers, unless their radii are equal.

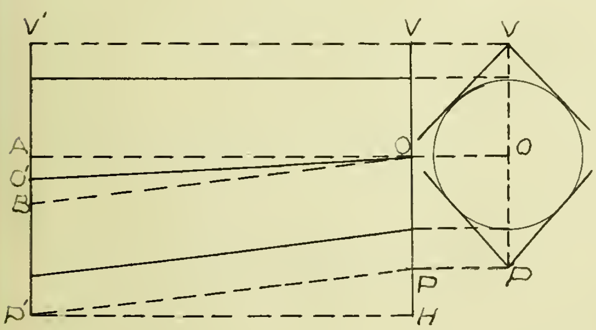


Fig. 4.

To determine the inequality of the pivots, let O and O', Fig. 4, be the centers of the west and east pivots, for clamp west; P and P', the vertices of

of the wyes in which the pivots rest; V and V,' the vertices of the wyes of the level; and HP', a horizontal line. Then O'OB = O'OA is the inequality of the pivots, which we shall represent by i. If we let

B' = the inclination given by the level for clamp west,

B'' = the inclination given by the level for clamp east,

b' = the true inclination for clamp west,

b'' = the true inclination for clamp east,

β = the constant angle, HP'P.

Then

$$b' = B' + i = \beta - i$$

for clamp west, and

$$b'' = B'' - i = \beta + i$$

for clamp east. Therefore

$$i = \frac{B'' - B'}{4}$$

The value of i was obtained by taking the mean of a number of determinations.

March 22, 1905

Clamp West		Clamp East		
W.	E.	W.	E.	B'' − B'
48.8	88.1	48.8	88.8	
76.7	37.5	75.0	35.3	
49.8	89.0	49.5	89.2	
16.8	31.7	14.8	35.3	
B' = −11.75d		B'' = −13.98d		−2.23d
49.7	88.1	50.7	89.6	
76.9	38.5	74.9	36.2	
49.7	88.0	50.7	81.3	
76.1	37.7	74.1	[illegible]	
B' = −11.58d		B'' = −14.85d		−3.27d
50.5	86.6	51.2	89.4	
75.6	37.1	13.7	35.6	
50.1	[illegible]	51.1	89.1	
74.9	37.2	73.4	35.5	
B' = −12.93d		B'' = −15.65d		−2.68d
51.4	89.0	51.5		
[illegible].6	31.8	73.3		
50.8	8[illegible]	51.7		
[illegible]4	37.[illegible]	73.3		
B' = −13.33d		B'' = −16.03d		−2.70d

Mean −2.72d.

$u = \frac{1}{4}(-2.72)(0''.367) = -0''.250 = -0^s.017$

7. Determination of the Collimation Constant —

The collimation constant is positive when the middle wire is actually ~~east~~ west of the collimation plane. It may be determined by observing the transit of a circumpolar star over a set of wires and then quickly reversing the instrument so as to observe the star's transit over the same set of wires again.

Let T_1 represent the time of transit of the star over a certain fixed wire and T_2, the time of transit over the same wire after the instrument has been reversed. The difference of the two transits ($T_2 - T_1$) multiplied by the cosine of the declination gives the equatorial time interval between the two transits. This reduction is made because the wire intervals which are involved in the discussion are given in equatorial time intervals. Now, it is quite evident that

$$T_0 = T_1 + (I + c)\sec\delta$$

and also that

$$T_0 = T_2 - (I + c)\sec\delta$$

By eliminating T_0, we obtain

$$c = \frac{T_2 - T_1}{2}\cos\delta - I$$

where T_0 represents the time of transit over the middle wire if there is no collimation and I the wire interval of the thread under consideration

Example:- The collimation constant was determined by means of observations made upon 43 H. Cephei at upper culmination and upon Groombridge 2001 at lower culmination on Nov. 23, 1904. As the sign of c is determined with respect to the position of the instrument at the time T_1, we find that the collimation constant is negative for both determinations. But as the instrument was clamp east in both cases at the time, T_1, the collimation constant would be positive for clamp west, the normal position of the instrument.

43 H. Cephei 85° 45′ (Clamp East)

Wire	T_1	T_2	$\frac{T_2-T_1}{2}$	$\log \frac{T_2-T_1}{2}$	$\log \cos\delta$	$\log \frac{T_2-T_1}{2}\cos\delta$	$\frac{T_2-T_1}{2}\cos\delta$	I	c
E_5	$00^h 51^m 02^s.0$	$01^h 01^m 12^s.7$	$305^s.35$	2.4848	8.8699	1.3547	22.63	23.12	-0.49
E_4	$51^m 22^s.5$	$00^m 50^s.9$	284.20	2.4536	8.8699	1.3235	21.06	21.47	-0.41
E_3	$51^m 47^s.7$	$00^m 25^s.9$	259.10	2.4135	8.8699	1.2834	19.20	19.74	-0.54
E_2	$52^m 08^s.9$	$00^m 06^s.9$	239.00	2.3784	8.8699	1.2483	17.71	18.15	-0.44
E_1	$52^m 31^s.8$	$00^h 59^m 42.1$	215.15	2.3327	8.8699	1.2026	15.94	16.50	-0.56

-0.49

Groombridge 2001 72° 53′ (Clamp East (lower culmination)

Wire	T_1	T_2	$\frac{T_2-T_1}{2}$	$\log \frac{T_2-T_1}{2}$	$\log \cos\delta$	$\log \frac{T_2-T_1}{2}\cos\delta$	$\frac{T_2-T_1}{2}\cos\delta$	I	c
E_5	$01^h 22^m 26^s.2$	$01^h 24^m 59^s.9$	$76^s.85$	1.8856	9.4688	1.3544	22.62	23.12	-0.50
E_4	$22^m 32^s.1$	$24^m 54^s.6$	71.25	1.8528	9.4688	1.3216	20.97	21.47	-0.50
E_3	$22^m 38^s.1$	$24^m 48^s.3$	65.10	1.8136	9.4688	1.2824	19.16	19.74	-0.58
E_2	$22^m 43^s.2$	$24^m 43^s.2$	60.00	1.7782	9.4688	1.2470	17.66	18.15	-0.49
E_1	$22^m 49^s.0$	$24^m 38.1$	54.55	1.7367	9.4688	1.2056	16.05	16.50	-0.45

-0.50

$c = + 0^s.50$ for clamp west.

The collimation constant may also be determined by sighting on a distant terrestrial object. The telescope must be placed in a horizontal position and some well-defined distant point, the image of which is near the middle wire, selected to be employed in the determination. The distances of the image from the middle wire in the two positions of the instrument are measured with the micrometer. This distance will be called D' for clamp west and D'' for clamp east. Since the eye-piece reinverts horizontally D' and D'', will be positive and negative according as the middle wire is east or west (in the eye-piece) of the image. The collimation constant is then given by the following equation

$$c = \frac{1}{2}(D' - D'')$$

for clamp west, the normal position of the instrument.

April 26, 1905

Clamp	C_3	micrometer on image	micrometer on C_3	Clamp	C_3	micrometer on image
W.	W. of image	23.320	23.180	E	W. of image	22.537
		23.517	23.085			22.728
		23.298	23.179			22.537
		23.512	23.081			22.729
		23.303	23.183			22.534
		23.514	23.079			22.729

Mean = 23.411 23.131 Mean = 22.632

$D' = -(23.411 - 23.131)R = -0.280 R.$

$D'' = -(23.131 - 22.632)R = -0.499 R.$

$c = \frac{1}{2}[-0.280 R - (-0.499) R] = +0.11 R = +0.^{s}41$

8. Determination of the Azimuth Constant — The only method by which this constant can be determined is by observations on the stars. Two stars must be observed; let these stars be represented by (α_1, δ_1) and (α_2, δ_2). After all the other constants have been determined, we can correct the times of observation of the two stars for all errors except the azimuth. Let T_1 and T_2 represent these corrected times. Then for the first star observed, we have

$$\Delta T = \alpha_1 - T_1 - a A_1$$

and for the second

$$\Delta T = \alpha_2 - T_2 - a A_2$$

where A_1 and A_2 are the values of the transit factor A corresponding to δ_1 and δ_2 respectively. Combining the above equations, we obtain

$$a = \frac{(\alpha_1 - T_1) - (\alpha_2 - T_2)}{A_1 - A_2}$$

In order to obtain an accurate determination of the azimuth constant, it is evident that all the other instrumental errors must be well determined. The value of a will be determined best when the denominator $A_1 - A_2$ is made as large as possible. This condition will be fulfilled by observing one star at lower culmination and the other at upper culmination, both as near the pole as possible.

Example:- For this determination, observations were made on 43 H Cephei and Groombridge 2001. As the instrument was reversed on each star, the collimation constant was eliminated and need not be considered.

Nov. 23, 1904

Star	T	Diab.	Bb	Sum	Corr'd T	α	Constants
43 H. Cephei	$00^h 56^m 7.^s14$	−0.22	+0.24	+0.02	7.16	$00^h 55^m 56.^s76$	$b = +0.025$
Gr. 2001 S.P.	$01^h 23^m 43.^s27$	+0.06	−0.03	+0.03	43.30	$01^h 23^m 36.^s97$	

Star	δ	B	A
43 H. Cephei	$+85°45'01''$	+9.44	−9.65
Gr. 2001	$+79°53'04''$	−1.33	+3.13

$\alpha_1 - T_1 = -6.33$ $\alpha_2 - T_2 = -10.40$

$$a = \frac{-6.33 + 10.40}{3.13 + 9.65} = 0.^s32$$

9. Clock Correction — As the relation, which exists between the time and the instrumental errors of a transit, has already been determined, it would be well at this point to illustrate the record and reduction of a set of transit observations by means of which an ordinary determination of time is obtained. The necessary data are the date; b, the measured inclination of the horizontal axis; and T, the time that the chronometer shows for the transit of the star. The observed times of transit given are each the mean of the observed times of transit of the given star over five wires and in the reduction the collimation constant, c, is assumed to refer to the mean of these wires. This assumption can be made because each star was observed over exactly the same set of wires

In our determination of the clock correction, we get $a = +0^s.37$, $c = +0^s.54$ and $\Delta T = -7^s.10$. Now, if we use the values of the constants, $a = +0^s.36$ and $c = +0^s.50$, obtained by direct determination on the same evening, we get $\Delta T = -7^s.09$. This comparison shows that all three quantities can be determined at the same time with fairly accurate results.

Clock Correction
Nov. 23, 1904

	Star	Clamp	T	Di. Ab.	A a	B b	C c	Sum	Corr'd T	α	ΔT	Constants
(1)	41 H. Cephei	W	$23^h 43^m 29^s.86$	$-0^s.04$	$-0^s.44$	$+0^s.05$	$+1^s.40$	$+1^s.00$	$43^m 30^s.86$	$23^h 43^m 23^s.72$	$-7^s.14$	$b = +0^s.038$
(2)	ω Piscium	W	$23^h 54^m 32^s.16$	$-0^s.02$	$+0^s.21$	$+0^s.03$	$+0^s.55$	$+0^s.77$	$54^m 32^s.93$	$23^h 54^m 25^s.86$	$-7^s.07$	$a = +0^s.37$
(3)	ι Ceti	E	$00^h 14^m 42^s.50$	$-0^s.02$	$+0^s.25$	$+0^s.02$	$-0^s.53$	$-0^s.27$	$14^m 42^s.23$	$00^h 14^m 35^s.11$	$-7^s.12$	$c = +0^s.54$
(4)	ξ Cassiop.	E	$00^h 31^m 49^s.86$	$-0^s.03$	$-0^s.14$	$+0^s.06$	$-0^s.90$	$-1^s.01$	$31^m 48^s.85$	$00^h 31^m 41^s.79$	$-7^s.06$	$\Delta T = -7^s.10$

	δ	B	A C
(1)	$+67° 17'$	$+2.30$	$\Delta T - 1.19a + 2.59c = -6.18$
(2)	$+6° 20'$	$+0.83$	$\Delta T + 0.56a + 1.01c = -6.31$
(3)	$-1° 21'$	$+0.66$	$\Delta T + 0.77a - 1.01c = -7.39$
(4)	$+57° 28'$	$+1.63$	$\Delta T - 0.39a - 1.68c = -8.10$

From
$(1) - (2) - (3) + (4)$

$-2.91a + 0.91c = -0.58$

From
$-(1) - (2) + (3) + (4)$

$1.01a - 6.29c = -3.00$

or

$a = +0.37 \quad c = 0.54$

Clock Correction
Nov. 23, 1904
(using values of a and c obtained by direct determination).

Star	Clamp	T	Di. Ab.	A a	B b	C c	Sum	Corr'd T	α	ΔT	Constants
41 H Cephei	W	$23^h 43^m 29^s.56$	$-0^s.04$	$-0^s.33$	$+0^s.05$	$+1.29$	$+0.95$	$43^m 30^s.81$	$23^h 43^m 23^s.72$	$-7^s.09$	$c = +0^s.50$
ω Piscium	W	$23^h 54^m 32^s.16$	$-0^s.02$	$+0.18$	$+0^s.03$	$+0.51$	$+0.70$	$54^m 32.86$	$23^h 54^m 25^s.86$	$-7^s.00$	$b = +0^s.038$
ι Ceti	E	$00^h 14^m 42^s.50$	$-0^s.02$	$+0.25$	$+0^s.02$	-0.51	-0.25	$14^m 42.25$	$00^h 14^m 35^s.11$	$-7^s.14$	$a = +0^s.32$
ξ Cassiop.	E	$00^h 31^m 49^s.56$	$-0^s.03$	-0.12	$+0^s.06$	-0.84	-0.93	$31^m 48^s.93$	$00^h 31^m 41^s.79$	$-7^s.14$	$\Delta T = -7^s.09$

10. Conversion of Time — If the clock correction for a chronometer keeping mean solar time is desired, it can only be found by comparing the chronometer, the clock correction of which is desired, with the sidereal chronometer, the clock correction of which has already been determined. Knowing the correction for the sidereal chronometer, we can convert sidereal time into the equivalent mean solar time and then by comparison find the clock correction for the chronometer keeping mean solar time. An illustration of the actual process is given below. The reader is referred to Comstock's "Field Astronomy for Engineers" for a complete explanation of the process of converting sidereal into mean solar time.

Example;— The Hornby chronometer kept sidereal time and the Bliss chronometer, standard time.

Nov. 23, 1904

Bliss Chronometer	=	$4^h\ 45^m\ 20^s.0$
Hornby Chronometer	=	$2^h\ 05^m\ 10^s.5$
$H\Delta T$	=	$-7^s.10$
Θ	=	$2^h\ 05^m\ 03^s.40$
Q_1	=	$16^h\ 09^m\ 04^s.43$
$\Theta - Q_1$	=	$04^h\ 55^m\ 08^s.97$
$(\Theta - Q_1)\ a''$	=	$01^m\ 37^s.90$
M		$09^h\ 53^m\ 31^s.07$
E. of 90^{th} Mer.	=	$07^m\ 06^s.09$
M (10^{th} Mer.)	=	$09^h\ 46^m\ 24^s.98$
$B\Delta T$	=	$+01^m\ 04^s.98$

11. <u>Precision of Results</u>:- If there were no errors made by the observer, we should always get the same value for the wire intervals when reduced to equatorial time interval. When this is done, many different values are obtained for the same wire interval. From these different values, the probable error of the value of a wire interval may be computed, as well as the probable error of a single transit. This will give some knowledge of the precision of the results.

Example:- The data was obtained from observations made on thirteen stars but only enough data will be given to show the arrangement and the method by which the probable error of a single transit is obtained.

Star	Wire	T	I	cos δ	I cos δ
o Andromedae	C_1	26.3	8.9	0.745	6.63
	C_2	22.4	5.0		3.72
	C_3	17.4			
	C_4	13.0	4.4		3.28
	C_5	8.7	8.7		6.48
ι^2 Aquarii	C_1	16.9	8.9	0.927	8.27
	C_2	11.0	4.0		3.72
	C_3	07.0			
	C_4	04.3	2.7		2.51
	C_5	01.0	6.0		5.57
ι Pisc. Austr.	C_1	17.0	8.1	0.865	7.01
	C_2	13.0	4.1		3.55
	C_3	08.9			
	C_4	05.1	3.8		3.29
	C_5	01.1	7.8		6.75
ι Andromedae	C_1	00.4	9.9	0.734	7.27
	C_2	05.1	5.2		3.82
	C_3	10.3			
	C_4	15.1	4.8		3.52
	C_5	11.3	9.0		6.61

Now, if we take the mean of the values found for each wire interval and form residuals, we can compute the probable error in the value of any one wire interval. Since the wire interval is merely the difference of time between the transits of the star over the given wire and the middle wire, we are able to find the probable error of any one transit

C_1		C_2		C_4		C_5	
$I\cos\delta$	v	$I\cos\delta$	v	$I\cos\delta$	v	$I\cos$	v
6.63	0.44	3.72	0.09	3.28	0.00	6.48	0.05
8.27	1.20	3.72	0.09	2.51	0.77	5.57	0.86
7.01	0.06	3.55	0.08	3.29	0.01	6.75	0.32
7.27	0.20	3.82	0.19	3.52	0.24	6.61	0.18
7.05	0.02	3.67	0.04	3.86	0.58	6.86	0.43
mean = 1.01	[+v] = 4.11	mean = 3.63	[+v] = 1.11	mean = 3.28	[+v] = 5.97	mean = 6.43	[+v] = 3.55

As we have fifty two residuals and four quantities involved

$$r_I = \frac{0.845\,(12.62)}{\sqrt{52 \times 48}} \quad \text{(probable error of a single}$$

wire interval)

The probable error of a single transit would be

$$r = \frac{0.845\,(12.62)}{\sqrt{2}\,\sqrt{52 \times 48}} = 0^s.15$$

The real test for accuracy and care in observing

is the agreement of the various values of ΔT determined from observations made on different stars on the same night. There was not enough data to compute the probable error from these results.

Neither this apparent precision nor the precision that would be obtained from comparing the different values of ΔT, is completely reliable, for nearly all observers possess individual peculiarities, called personal equation, which cause them to observe all stars either too soon or too late by an almost constant amount. No indication of the presence or magnitude of this constant personal error can be detected by determining the probable error of a single transit. The personal equation, although often a considerable source of error, is of no practical consequence unless the observations of different persons are to be combined. In such cases, the personal equation of each observer must be determined.

Various devices are employed to determine the amount of the personal equation of an observer. In this determination, a so-called personal-equation machine was used. A fine point of light moved back and forth across a set of wires and its transits were recorded automatically by a chronograph. The observer also records the transits by means of a key connected with the chronograph. The difference between the two records was measured and in that way, the personal equation was determined.

Twenty sets of observations were made but only

a few will be given to show the arrangement of the data and the method of the determination. The probable error of the personal was determined in order to see how reliable the value of the personal equation really was.

Wed. Dec. 14, 1904

R to L	v	L to R	v	R to L	v	L to R	v
1.09	9	1.15	8	1.11	1	0.95	6
1.10	8	1.10	3	1.09	3	1.20	19
1.22	4	1.11	4	1.15	3	1.06	5
1.16	2	0.98	9	1.12	0	0.84	17
1.33	15	1.00	7	1.12	0	1.02	1
1.18	38	1.068	31	1.118	7	1.014	48
	0.112				0.104		
P.E.	0.056			P.E.	0.052		

In this way twenty values were obtained for the personal equation and by taking their mean

Personal Equation = $0^s.06$ Late

and found that the

Probable error of the personal equation due to any one set

$$= \frac{0.8453 \times 0.\ldots1}{\sqrt{20 \times 19}} = 0.013 \quad (0.0127)$$

Probable error of the result = $\frac{0.013}{\sqrt{20}} = 0.003$

Then by forming residuals for each single observation in a set, we find that the

Probable error of any single observation = $\frac{0.845 \times 8.78}{\sqrt{200 \times 160}} = 0.04$

Probable of any single transit = $\frac{0.04}{\sqrt{2}} = 0.029$

Probable error of the mean of any set $= \frac{0.04}{\sqrt{5}} = 0.019$

Probable error of the difference between any pair of means

$$= \sqrt{2} \times 0.019 = 0.0266$$

Probable error of Personal Equation due to any ~~set~~

$$= \frac{0.0266}{2} = 0.013 \qquad (0.0133)$$

Probable error of result $= \frac{0.013}{\sqrt{20}} = 0.003$

The probable error of the result was found to be the same by two methods of computation, therefore

$$\text{Personal equation} = 0^s.06 \pm 0^s.003 \text{ (late)}$$

12. Zenith Telescope - Description - As the transit instrument has a sensitive spirit level attached at right angles to the rotation axis, and the micrometer box can be rotated so that the micrometer wire moves parallel to this axis, it is also a zenith telescope. The level is called a zenith level.

13. Determination of Geographical Latitude - The latitude may be determined quite accurately by means of the zenith telescope. Talcott's method is the one employed and consists in measuring the difference of the zenith distances of two stars, one of which culminates south of the zenith and the other north of the zenith. This difference should not be much larger than one half the diameter of the field of view, to avoid observing too near the edge of the field. The difference of the right ascensions of the two stars should not exceed twenty minutes, to avoid any change in the constants of the instrument between the two halves of the observation, and should never be so small as to cause undue haste. The zenith distances should always be less than 35° to avoid uncertainty in refraction.

If we let δ', z' and δ'', z'' be the declinations and zenith distances of the south and north star respectively, then

$$\delta' = \Phi - z'$$

$$\delta'' = \phi + z''$$

Therefore

$$\delta' + \delta'' = 2\phi + (z'' - z')$$

where $z'' - z' < 40'$, is the condition which the pair of stars must fulfill for this instrument.

In this particular case, the approximate latitude was $40°06'$ and the instrument's field of view was $60'$ in diameter. The form of the observing list was, as follows:

Star	Mag	AR	δ	z	Setting	micrometer
λ Ursae Maj	3.3	$10^h 11^m 19^s$	$+43°24'$	$03°18'$	N $29°16'$	$13'$ up
31 Leon min.	4.3	$10^h 22^m 20^s$	$+37°12'$	$02°54'$	S $29°16'$	$13'$ down

To observe a star, the "setting" is made as described for the use of the transit, then the zenith level is unclamped and rotated until the bubble plays and then it is re-clamped. The bubble is brought near the middle of the tube by means of a screw at one extremity of the tube. Then, the micrometer wire is moved to the part of the eye-piece, where it is known the first star will pass. At the time of culmination or within a few seconds of this time, bisect the star with the micrometer wire and read the zenith level and the micrometer. Reverse the instrument gently, bring the bubble back to the center again by means of the tangent movement and observe the second star in the same manner as the first.

As the telescope must be shifted in a majority of cases when the instrument is reversed, extreme care was required to prevent a change in the angle between the position of the level and the line of sight during the progress of an observation. This angle must be preserved.

Let m_0, be the micrometer reading of some point in the field assumed as the micrometer zero: z_0 the apparent zenith distance for the reading m_0, when the bubble is at the center of the zenith level tube; m', m'' the micrometer readings on the two stars, the readings being supposed to increase with the zenith distance; R, the value of a revolution of the micrometer screw; b', b'' the level constants for the two stars, plus when the north end is high. The true zenith distance of the southern star is then

$$z' = z_0 + (m' - m_0) R + b' + r' ;$$

and of the northern star

$$z'' = z_0 + (m'' - m_0) R - b'' + r''$$

Since

$$\delta' + \delta'' = 2\Phi + (z'' - z')$$

we obtain by solving for Φ

$$\Phi = \tfrac{1}{2}(\delta' + \delta'') + \tfrac{1}{2}(m' - m'')R + \tfrac{1}{2}(b' + b'') + \tfrac{1}{2}(r' - r'')$$

As the zero of the level scale is at one end of the tube

$$\tfrac{1}{2}(b' + b'') = \tfrac{1}{2}\left[\pm(n' - s'') \pm (s' - n'')\right] d$$

the upper sign being used when n' is greater than s', the lower when the reverse is true.

The refraction correction is small and may be taken from tables which have been computed for this purpose. The following table, due to Campbell, was employed by the writer. The sign of this correction is the same as that of the micrometer correction

Values of $\frac{1}{2}(r'-r'')$

$\frac{z'-z''}{2}$	$z=0°$	$z=10°$	$z=20°$	$z=25°$	$z=30°$	$z=35°$
0'	$''.00$	$''.00$	$''.00$	$''.00$	$''.00$	$''.00$
1'	.02	.02	.02	.02	.02	.02
2'	.03	.03	.04	.04	.04	.05
3'	.05	.05	.06	.06	.07	.08
4'	.07	.07	.08	.08	.09	.10
5'	.08	.09	.10	.10	.11	.13
6'	.10	.10	.11	.12	.13	.15
7'	.12	.12	.13	.14	.15	.18
8'	.13	.14	.15	.16	.18	.21
9'	.15	.16	.17	.18	.20	.23
10'	.17	.18	.19	.21	.23	.26
11'	.18	.19	.21	.23	.25	.28
12'	.20	.21	.23	.25	.27	.31

The arrangement and reduction of the data are given on the next page. $R = 56''.28$ and $d = 0''.357$

Star	Level N	Level S	R		δ
o Ursae maj	21.3	63.8	62.150		61° 02' 13".1
δ Cancri	12.1	3[illegible].0	19.784		18° 30' 03.1
	+8.50		42.166	δ' + δ''	19° 32' 16.2
				½ (b' + b'')	+ 06.1
				mic	+40 18.1
				Ref'n	+ 00.8
				2 φ	80° 12' 41.2
				φ	40° 06' 20".6
36 Lyncis	39.1	66.8	24.543		43° 36' 34".1
38 Lyncis	10.0	62.5	63.296		37 12 14.8
	+23.0		38.153	δ' + δ''	80 48 48.9
				½ (b' + b'')	+ 16.4
				mic	−36 21.0
				Ref'n	− 0.6
				2 φ	80 12 43.3
				φ	40° 06' 21".65
λ Ursae maj	69.8	40.0	55.610		43° 23' 18".0
31 Leon min	33.9	63.8	31.856		37° 11' 36".5
	+6.05		23.154	δ' + δ''	80° 34' 54".5
				½ (b' + b'')	+ 4.3
				mic.	−22 16.9
				Ref'n	− 0.4
				2 φ	80° 12' 41".5
				φ	40° 06' 20".75
35 H Ursae maj	21.8	66.8	28.581		69° 34' 21".6
ι Leonis	64.8	62.5	54.[illegible]11		11° 02' 42".7
	+12.8		26.284	δ' + δ''	80° 37' 10".3
				½ (b' + b'')	+ 9.1
				mic	−24 39.2
				Ref'n	− 0.7
				2 φ	80° 12' 39.5
				φ	40° 06' 19.15

Mean φ = 40° 06' 20".1

Previous determination = 40° 06' 20".1

14 Reduction from Mean to Apparent Place — It is often found necessary to reduce the mean place of a star to apparent place before it may be employed for certain determinations. The mean places of stars are given in the Berliner Astronomisches Jahrbuch. Their apparent places may be calculated by the method given in the American Ephemeris, pp. 566-567.

Apparent places, April 11, 1905

	36 Lyncis	38 Lyncis	35 H Ursae Maj.
α_0	136° 54′	138° 01′	159° 04′
G	12° 46.′4	12° 36.′4	72° 43.′7
$G+\alpha_0$	209° 40.′4	210° 31.′4	231° 47.′7
$H'+\alpha_0$	383° 44.′4	384° 51.′4	405° 41.′7
δ_0	43° 37.′0	37° 12.′0	69° 34.′0
log g	0.13414	0.93494	0.93500
log cos$(G+\alpha_0)$	1.93895 n	9.93411 n	1.19132 n
log (g')	0.81381 n	0.86911 n	0.72632 n
log h	1.27855	1.27855	1.21859
log cos$(H+\alpha_0)$	1.96160	9.95118	9.84338
log sin δ_0	1.83764	9.78152	1.91179
log (h')	1.07884	1.01785	1.09376
log i	0.8793 n	0.8193 n	0.8190 n
log cos δ_0	1.85977	9.90118	9.54284
log (i)	0.13101 n	0.18048 n	0.42184 n
δ_0	43° 36′ 35.″02	37° 12′ 17.80	69° 34′ 23.″12
(g')	− 7.48	− 1.41	− 5.″33
(h')	+ 11.11	+10.42	+ 12.41
(i)	− 5.48	− 6.03	− 2.64
δ	43° 36′ 34.″05	37° 12′ 14.″78	69° 34′ 21.″56
; $'H$ =	246° 50.′4	246° 50.′4	246° 43.′7

References

Berliner astronomisches Jahrbuch, 1905 Pp. 149-312.

American Ephemeris, 1905 Pp. 210-400, 566-569.

Campbell - "Elements of Practical Astronomy" Pp. 65-86, 122-173

Chauvenet - "Astronomy" Pp. 131-282, 340-361

Comstock - "Field Astronomy for Engineers" Pp. 40-44, 99-111, 155-195.

Davis - Popular Astronomy, 1902 Pp. 303-308.